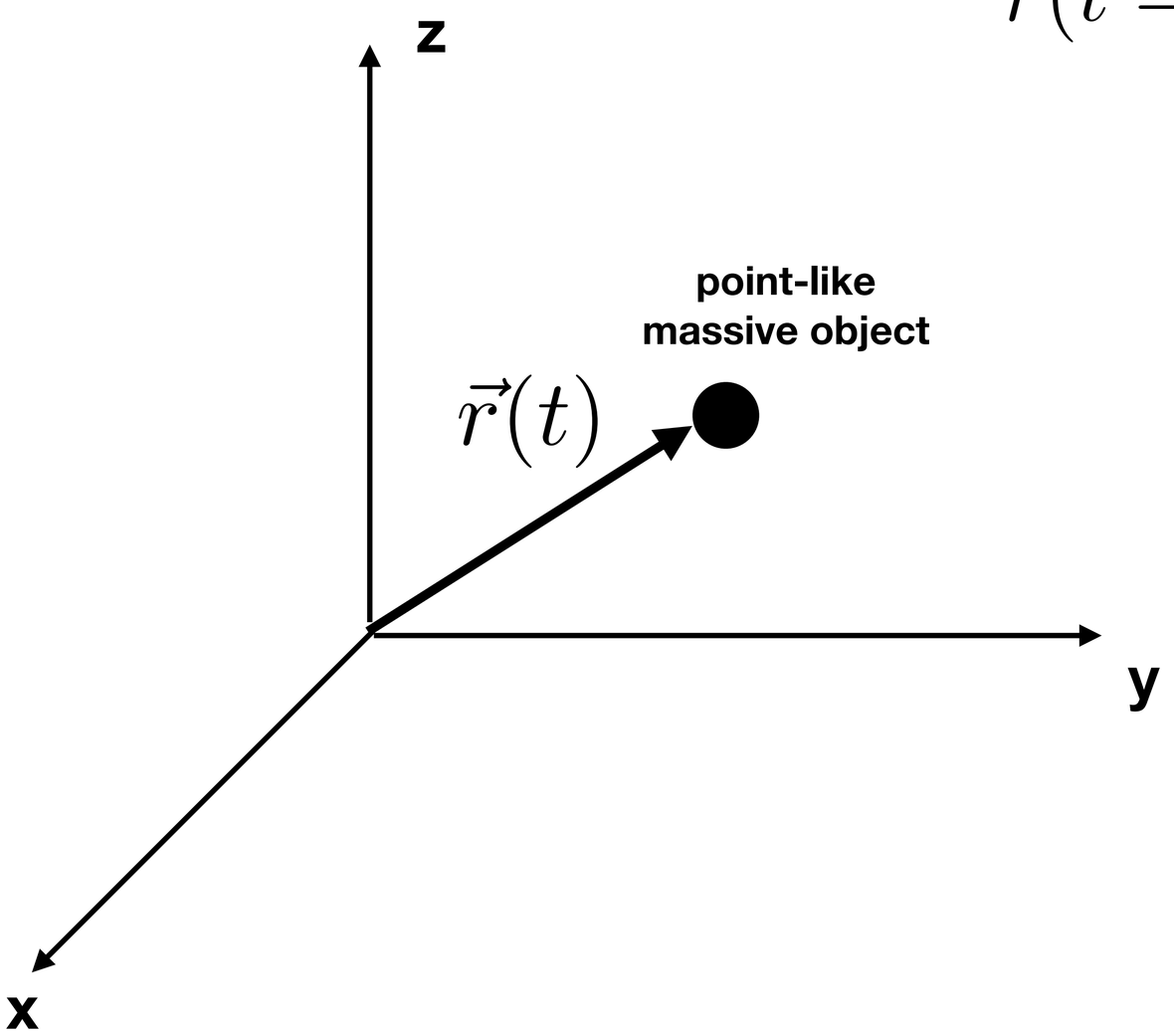
Phys 344: from classical to quantum mechanics

Sept 29/2025

Hamiltonian formalism of classical mechanics: brief summary



$$\vec{r}(t=0), \dot{\vec{r}}(t=0) \longrightarrow \vec{r}(t), \dot{\vec{r}}(t)$$

Hamiltonian H(r,p) is the total energy
It is provided in the system's ``user manual"

Solve 1st order differential equations

$$\dot{\vec{p}} = -\partial H(\vec{r}, \vec{p})/\partial \vec{r}$$
$$\dot{\vec{r}} = \partial H(\vec{r}, \vec{p})/\partial \vec{p}$$

Check $\mathrm{d}H(\vec{r}, \vec{p})/\mathrm{d}t = \partial H/\partial t$

Energy is a **conserved** quantity (unless you add or remove it explicitly)

Example 1: harmonic oscillator

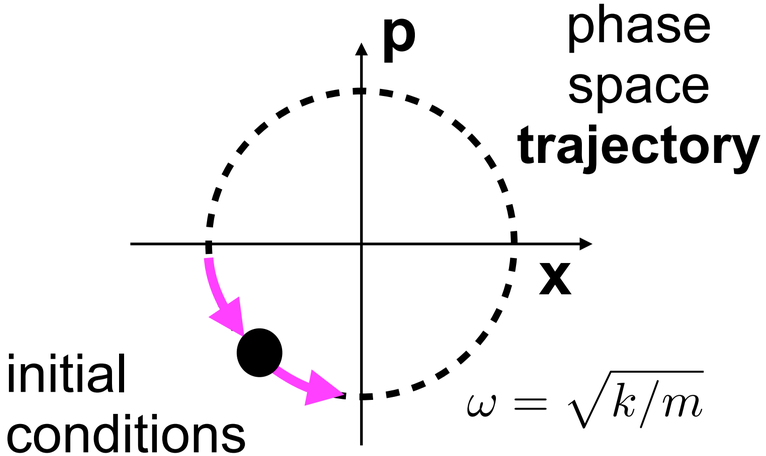
$$H(x,p) = p^2/2m + kx^2/2$$ ← **potential** energy

↑ **kinetic** energy

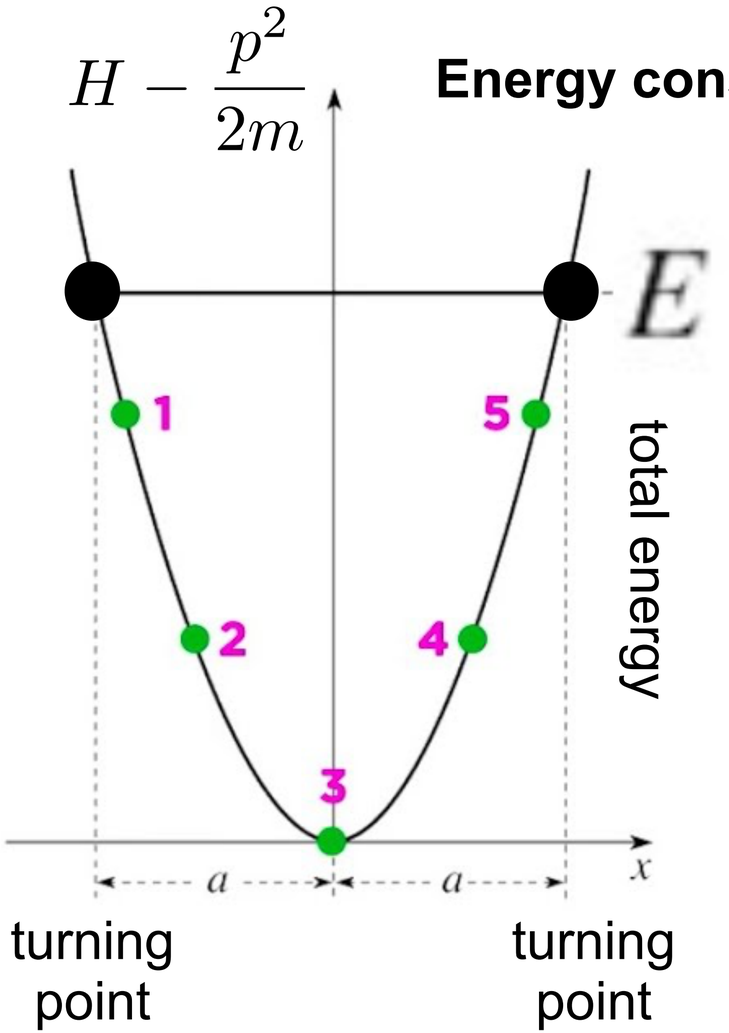
$$\begin{cases} \dot{x} = p/m \quad p = m\dot{x} \\ \dot{p} = -kx \end{cases}$$

here **p** is **momentum**

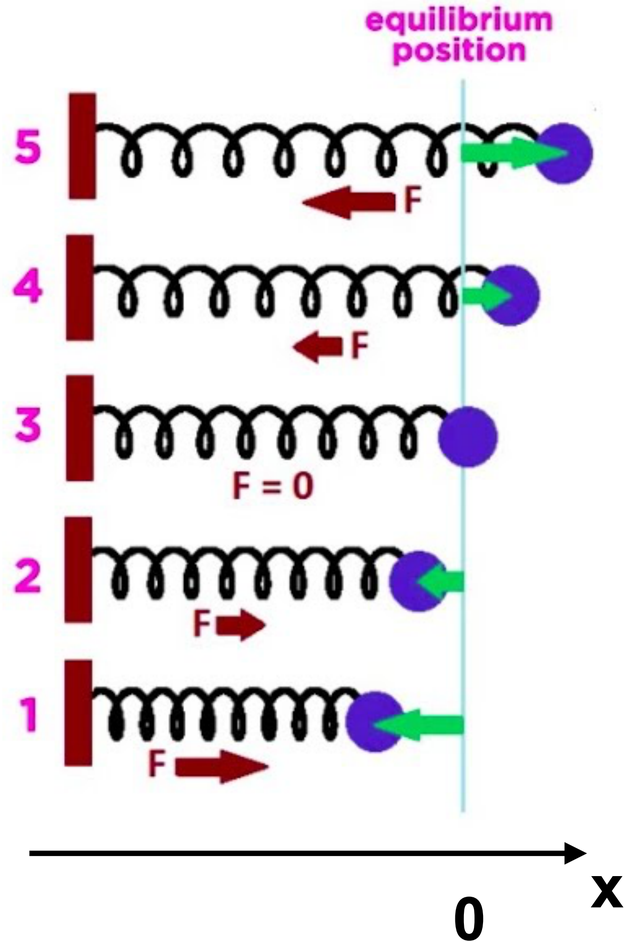
force



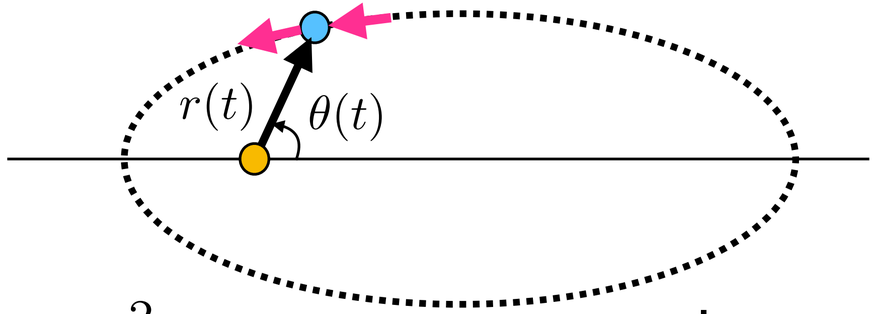
phase space **trajectory**



Energy conservation analysis



Example 2: central field motion (Sun & Earth, proton & electron)



real space trajectory

$$H - \frac{p^2}{2m}$$

$$H(r,p;\theta,\cancel{M}) = \frac{p^2}{2m} + \frac{L^2}{2mr^2} - \frac{k}{r}$$

L

Come with the user manual!

$m \approx 10^{-30}$ [kg]

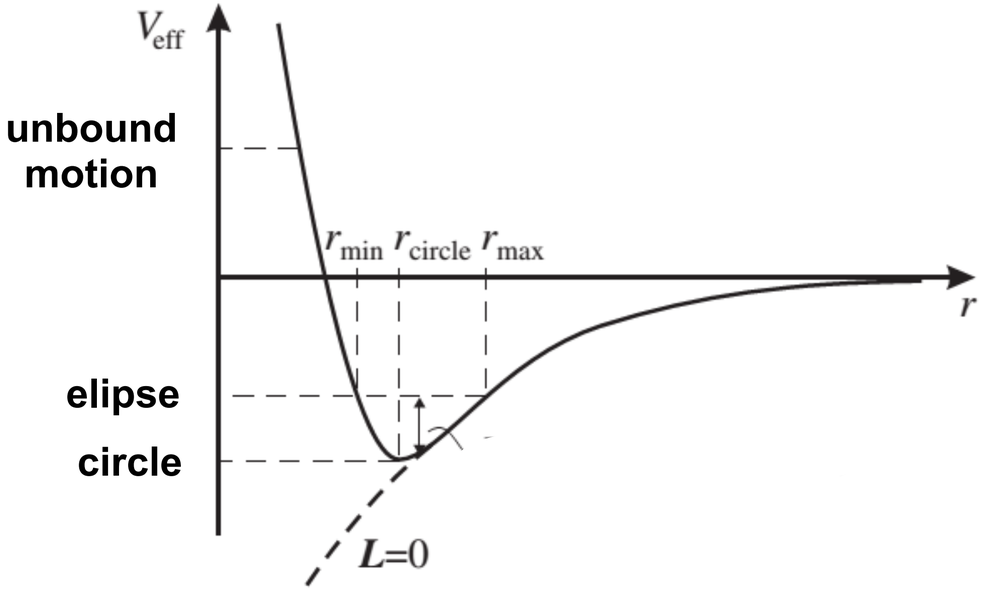
$k \approx 2.3 \times 10^{-28}$ [Joule] $\times$ [meter]

$$\dot{\theta} = \frac{\partial H}{\partial L} = L/mr^2 \quad L = mr^2\dot{\theta}$$

Angular momentum (**Spin**)

$$\dot{L} = -\partial H/\partial\theta = 0$$

Spin is cobserved under a central field potential



Circular electron-proton orbit:

$r \approx 10^{-10}$ [meter]

size of atoms

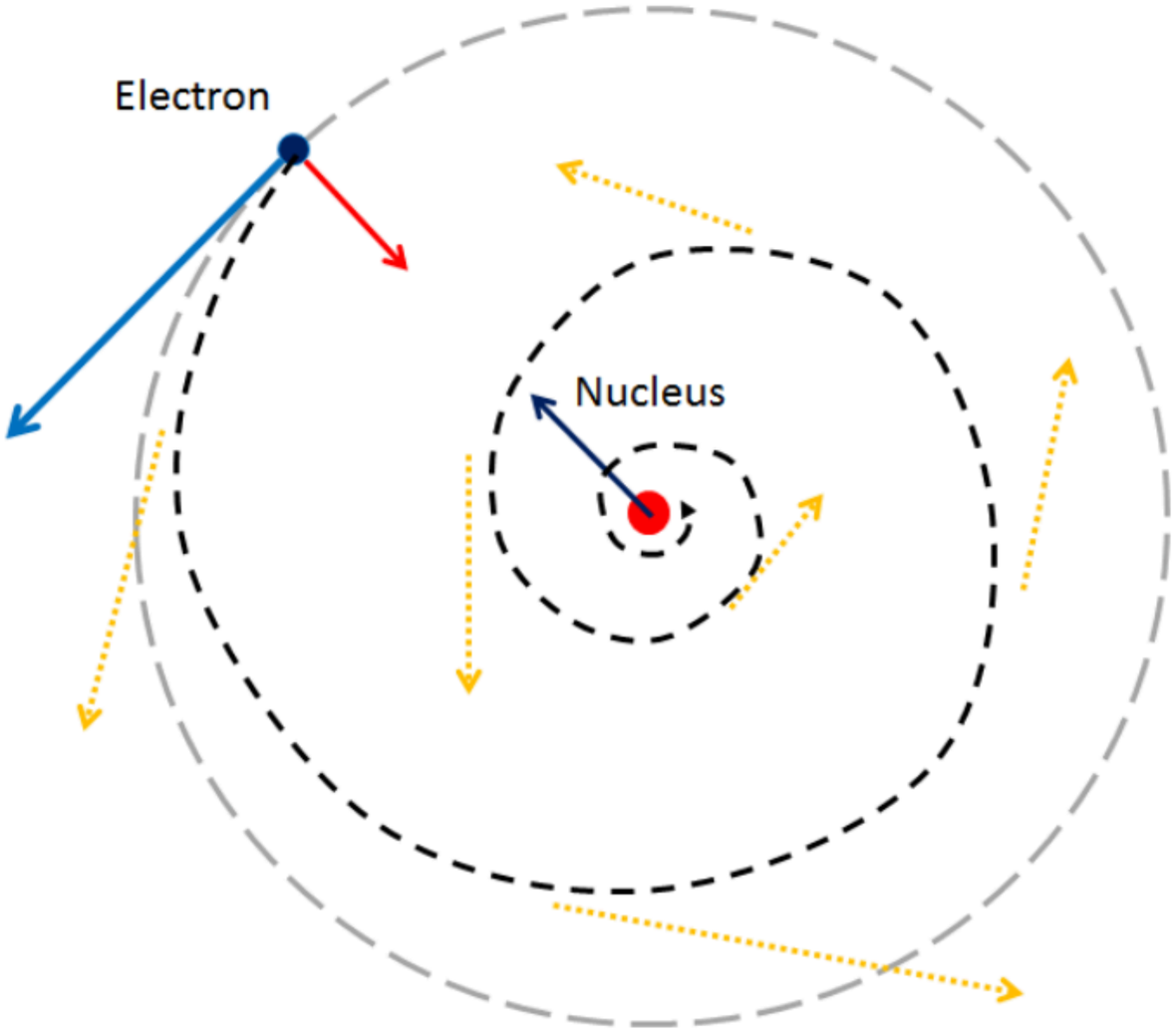
$L = \sqrt{mkr} \approx 10^{-34}$ [Joule] $\times$ [second]

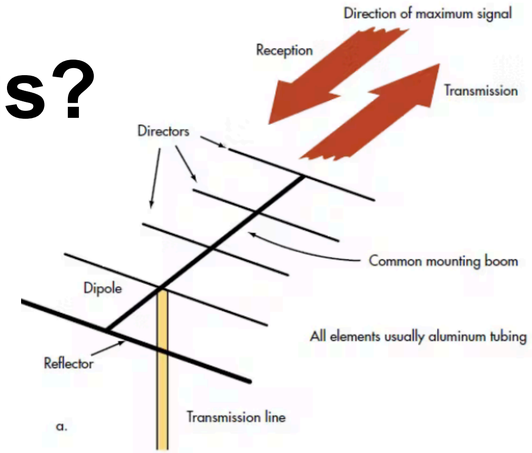
$\dot{\theta} = \omega = \sqrt{k/mr^3} \approx 2\pi \times (2 \times 10^{15})$ [Hertz] **Fast!**

$E = -k/2r \approx 10^{-18}$ [Joule]

$[E] = [L][\omega]$

How quickly will electron fall onto the nucleus?





$$\frac{dE}{dt} \approx -\frac{k\omega^4 r^2}{c^3}$$

dipole antenna radiation formula

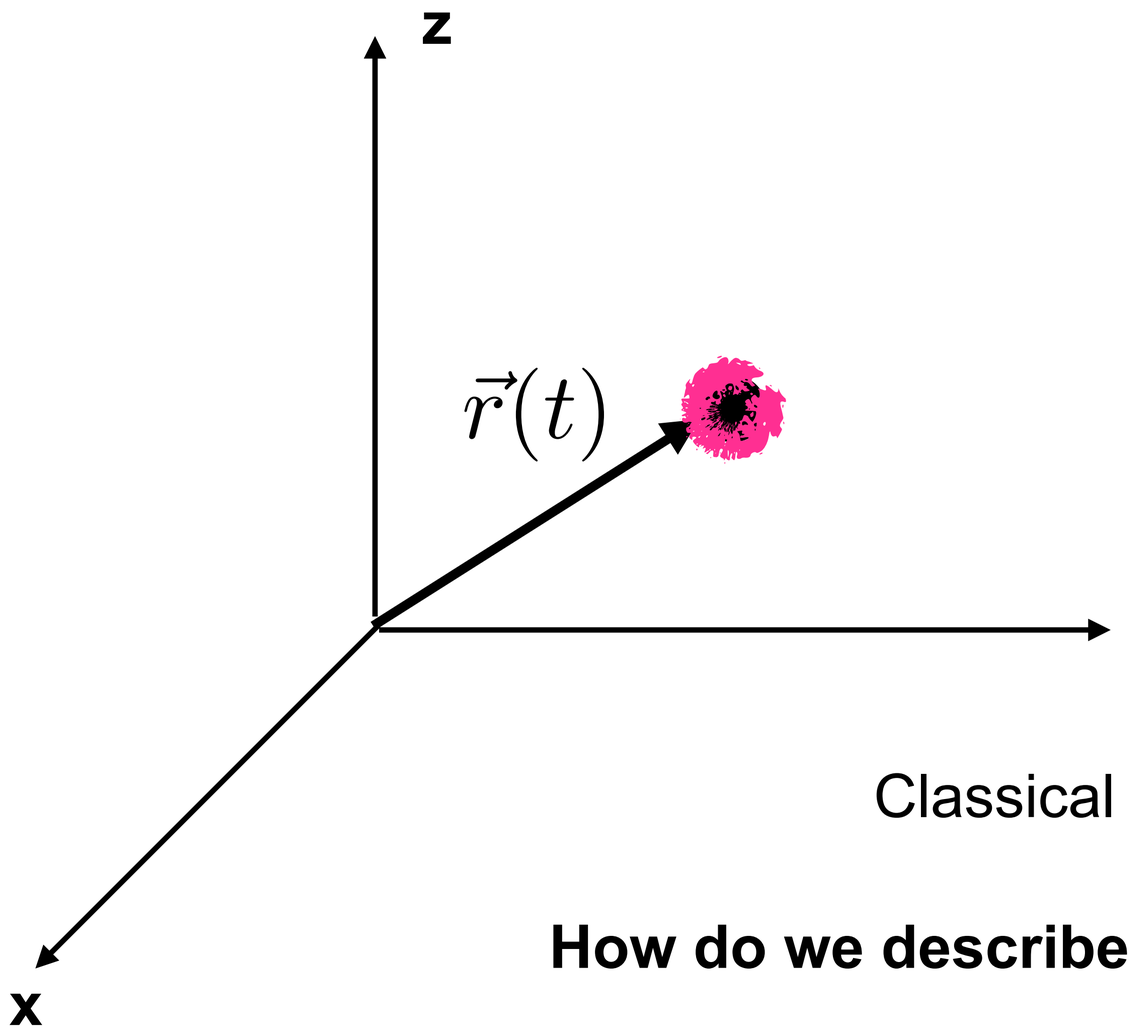
$c \approx 3 \times 10^8$ [meter]/[second]

$$E \approx -k/r$$

$$t \approx \frac{(c/r)^3}{\omega^4} \approx 10^{-9} \text{ [second]} = 1 \text{ ns}$$

Not good…

Enters Quantum mechanics



$$\vec{r}(t=0), \dot{\vec{r}}(t=0) \longrightarrow \vec{r}(t), \dot{\vec{r}}(t)$$

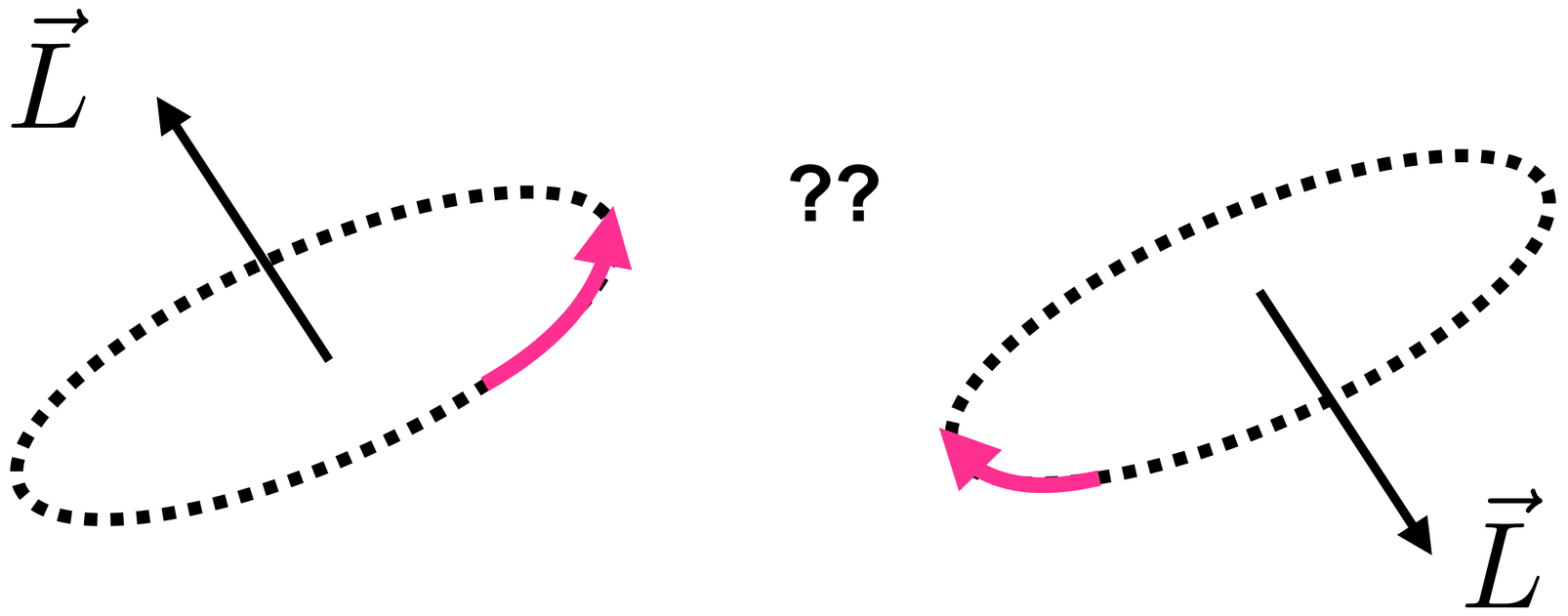
Impossible to ask where a particle is
AND
where it is going!

Classical mechanics recipe not applicable. Atoms are saved!

How do we describe the motion of quantum particles?

Step back: what questions are we allowed [by nature] to ask?

What is the direction of the **spin**?



We don’t even ask where the electron is or where it is going…
We merely ask if electron spins [spin is conserved] clockwise or anticlockwise?

A **spinning charge** follows the **magnetic** field **gradient**

Lorentz force

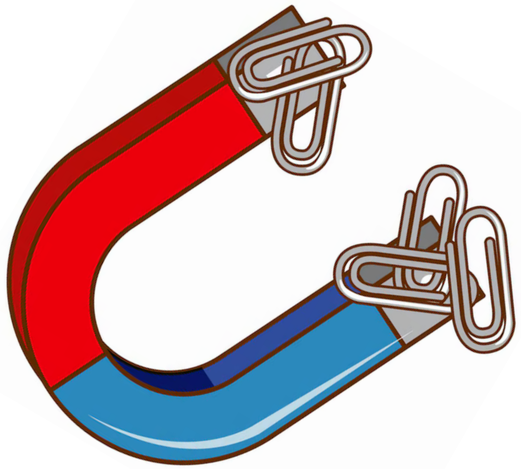
$F = e\omega r B$

$F = e\omega r B$

Avergae force if the spin is clockwise

B

magnetic field grows in this direction

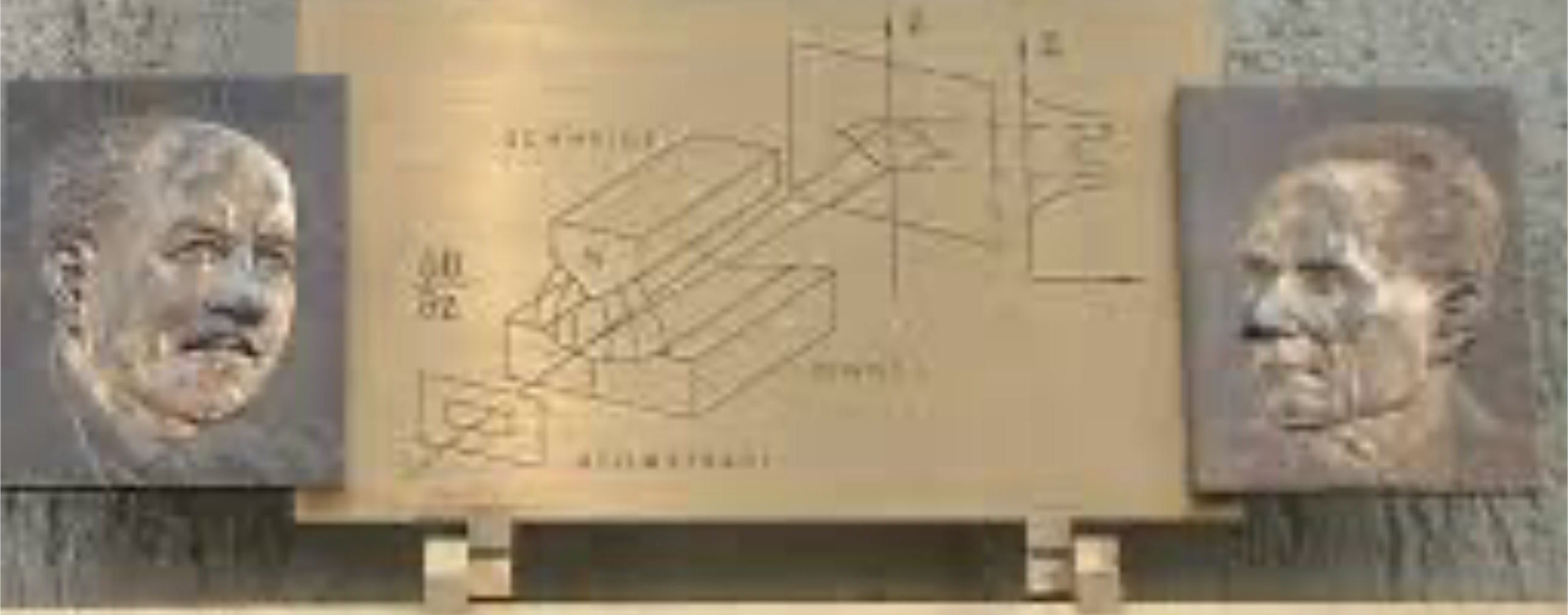


Avergae force if the spin is counterclockwise

$F = e\omega r B$

$F = e\omega r B$

$$\dot{x} = \frac{e\omega r^2}{m} \times \frac{dB}{dx} \approx 10^6 \text{ [meter]/[second]}^2$$

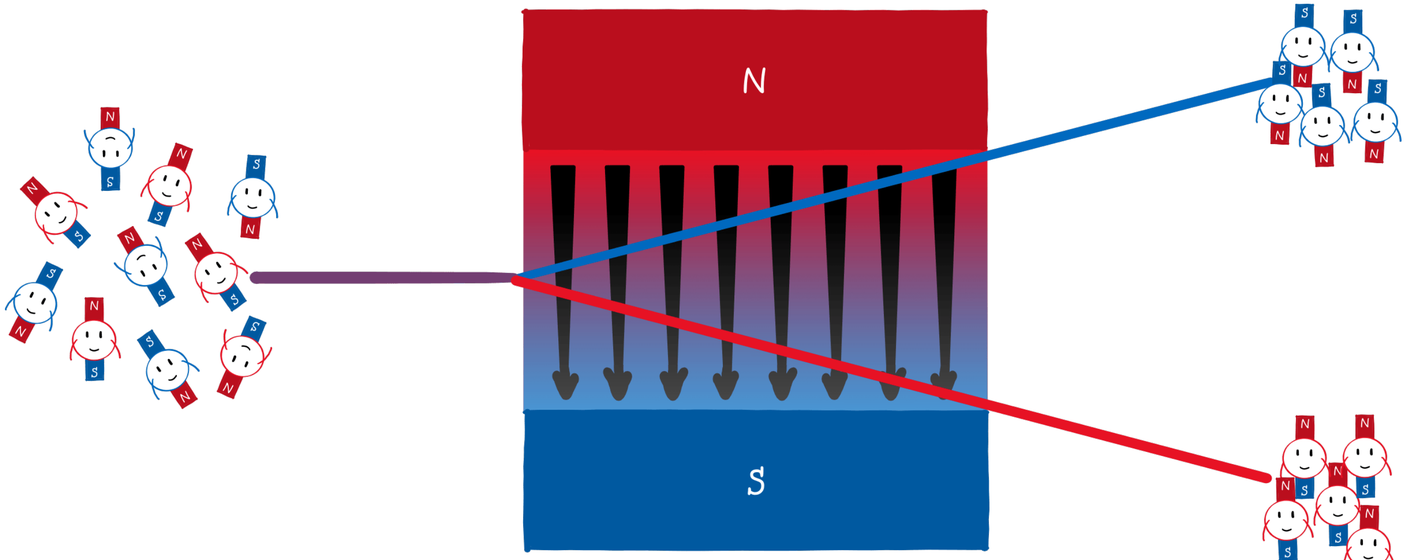


IM FEBRUAR 1922 WURDE IN DIESEM GEBÄUDE DES
PHYSIKALISCHEN VEREINS, FRANKFURT AM MAIN,
VON OTTO STERN UND WALTHER GERLACH DIE
FUNDAMENTALE ENTDECKUNG DER RAUMQUANTISIERUNG
DES MAGNETISCHEN MOMENTS IN ATOMEN GEMACHT

Stern-Gerlach experiment:

The spin did not know its orientation, until **asked**…!

analog process



spin-position entanglement

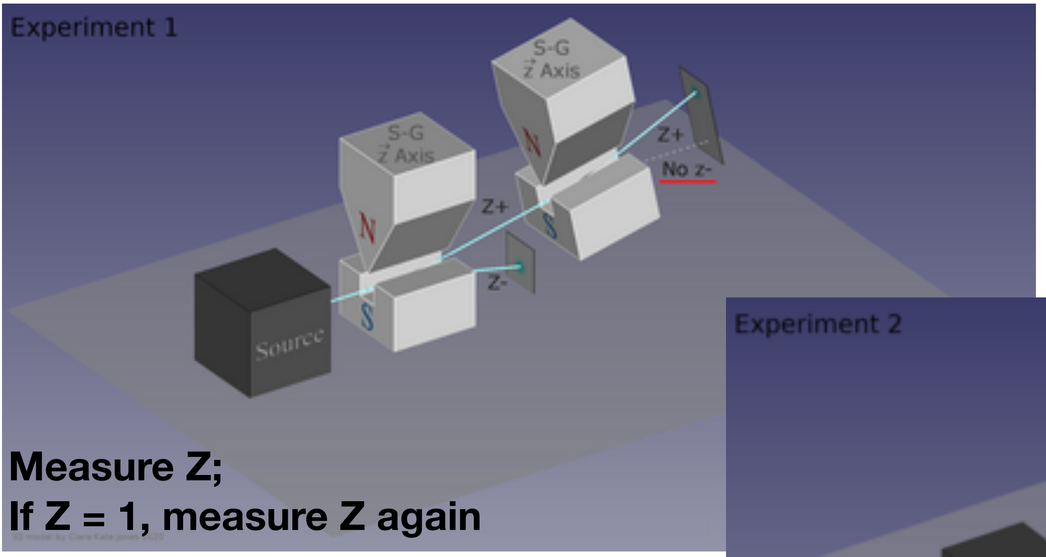
measuring spin orientation by measuring position

digital output

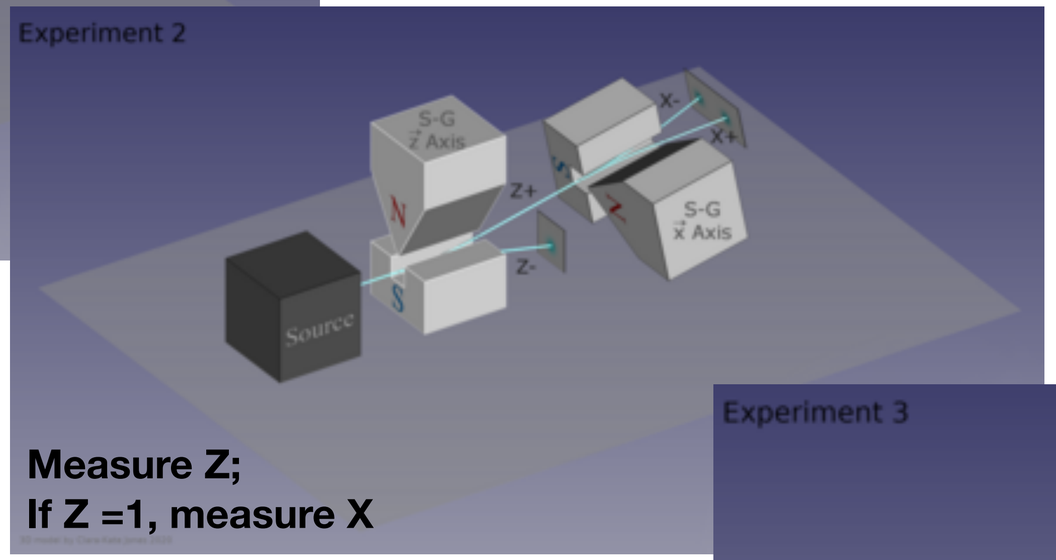
The answer is a) discrete and b) linked to the question

“Prompt engineering” gives even more strange outcome

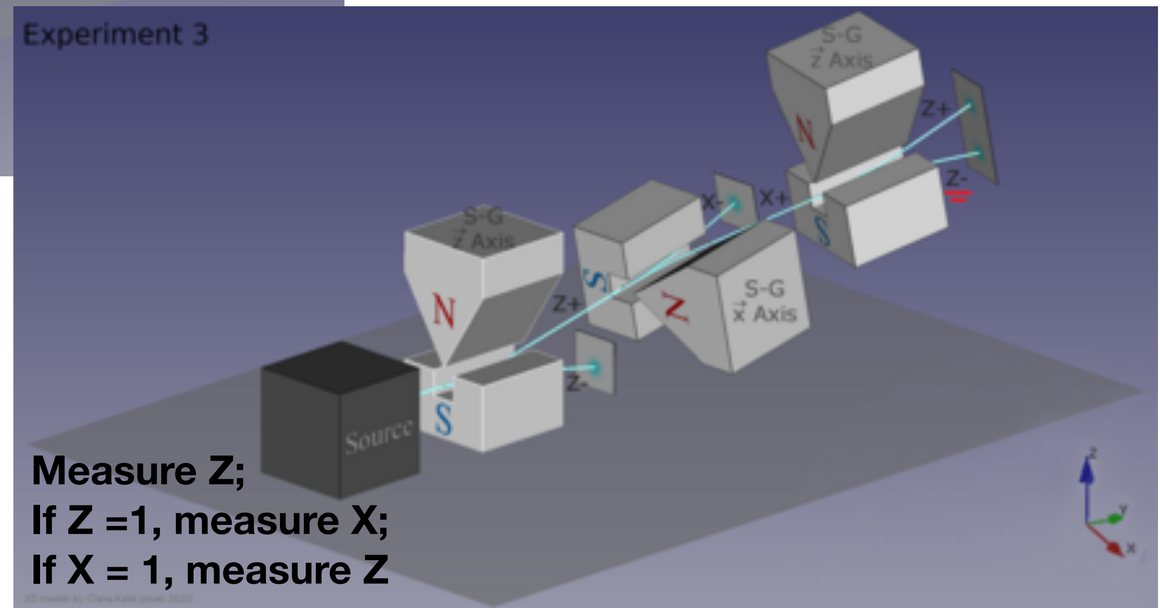
Feynman Lectures on Physics, vol 3 for a detailed discussion



Measure Z;
If Z = 1, measure Z again



Measure Z;
If Z =1, measure X



Measure Z;
If Z =1, measure X;
If X = 1, measure Z

Rotating the magnet changes the measurement axis in the querry of the spin projection value

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Explaining THAT requires a paradigm change for describing physics

COMING NEXT:
- vector spaces
- linear operators
- measurement rules
- quantum expectation values
- from 2D Hilbert space to 3D real space: the Bloch Sphere